Continued Implicit Relation and Common Fixed Point Theorems in Complex Valued Metric Space

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Abstract: In this paper we will prove some new common fixed point theorem s for generalized contractive maps in common valued metric space by using (E.A)and common property (E.A.)[1]satisfying an implicit relation which unify and generalize most of the existing relevant fixed point theorems using an implicit relation [2,8,9]

Key words: Compatible mappings, implicit relation, complex valued metric space, property (E.A), common property.

1. Introduction

The study of metric spaces expressed the most important role to many fields both in pure and applied science such as biology, medicine, physics and computer science (see [3]). Many authors

A. Azam, B. Fisher and M. Khan [4] first introduced the complex valued metric spaces which is more general than well-known metric spaces and also gave common fixed point theorems for maps satisfying generalized contraction condition.

The concept of weakly commuting mappings of Sessa [5] is sharpened by Jungck [6] and further generalized by Jungck and Rhoades [7]. Similarly, noncompatible mapping is generalized by Aamri and Moutawakil [1] called property (E.A) which allows replacing the completeness requirement of the space with a more natural condition of closedness of the range. There may be pairs of mappings which are noncompatible but weakly compatible). Let A and S be two self-maps of a metric space (X, d). Mappings A and S are said to be weakly commuting [3] if

(1.1) $d(SAx, ASx) \leq d(Ax, Sx)$, for all $x \in X$,

compatible [4] if

(1.2)$\lim_{n\to\infty} d(ASx_n, SAx_n) = 0$, whenever there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} A x_n = \lim_{n\to\infty} Ax_n = t$, for some $t \in X$. noncompatible if there exists a sequence$\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$, for some $t \in X$ and

(1.3)$\lim_{n\to\infty} d(ASx_n, SAx_n)$ is either nonzero or nonexistent, and weakly compatible if they commute at their coincidence points, i.e., ASu = SAu(1.4)whenever Au = Su, for some u ∈ X.

2. Definitions

Let us recall a natural relation on $\mathbb{C}$,for $z_1,z_2 \in \mathbb{C}$, define a partial order $\precsim$ on $\mathbb{C}$ as follows;

$z_1 \precsim z_2$ iff $Re(z_1) \le Re(z_2)$, $Im(z_1) \le Im(z_2)$

it follows that

$z_1 \precsim z_2$

if one of the following conditions is satisfied:

i. $Re(z_1)=Re(z_2)$, $Im(z_1)<Im(z_2)$
ii. $Re(z_1)<Re(z_2)$, $Im(z_1)=Im(z_2)$
iii. $Re(z_1)<Re(z_2)$, $Im(z_1)<Im(z_2)$
iv. $Re(z_1)=Re(z_2)$, $Im(z_1)=Im(z_2)$

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one the above conditions is not satisfied and we will write $z_1 \prec z_2$ if only iii is satisfied. Note that
$0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1|<|z_2|$,

$z_1 \precsim z_2$, $z_1 \prec z_2 \Rightarrow z_1 \prec z_3$

Definition 2.1let X be a nonempty set. A mapping d:XxX→ $\mathbb{C}$ is called a complex valued metric on X if the following conditions are satisfied:

(CM1) $0 \precsim d(x,y)$ for all $x,y \in X$ and $d(x,y)=0 \Leftrightarrow x=y$.

(CM2) $d(x,y)=d(y,x)$ for all $x,y \in X$

(CM3) $d(x,y) \precsim d(x,z)+d(z,y)$ for all $x,y,z \in X$.

In this case,we say that (X,d) is a complex valued metric space

Definition 2.2 Let $\mathbb{C}$ be a complex valued metric space,

- We say that a sequence $\{x_n\}$ is said to be a Cauchy sequence be a sequence in x ∈X If for every $c \in \mathbb{C}$, with $0 \prec c$ there is $n_0 \in \mathbb{N}$such that for all $n>n_0$ such that$d(x_n,x_m) \prec c$.
- We say that a sequence $\{x_n\}$ converges to an element $x \in X$. If for every $c \in \mathbb{C}$, with $0 \prec c$ ther exist an integer $n_0 \in \mathbb{N}$such that for all $n>n_0$ such that $d(x_n,x) \prec c$ and we write $x_n \underset{d}{\rightarrow} x$.
- We say that (x,d) is complete if every Cauchy sequence in X converges to a point in X.

Lemma 2.4 Any sequence $\{x_n\}$ in complex valued metric space (X,d), converges to x if and only if $|d(x_n,x)| \rightarrow 0$ as $n \rightarrow \infty$

Lemma 2.6 Any sequence $\{x_n\}$ in complex valued metric space (X,d) is a Cauchy sequence if and only if $|d(x_n,x_{n+m})| \to 0$ as $n \to \infty$, where $m \in \mathbb{N}$

Definition 2.3 Two self-maps s,T of a non-empty set X are said to be weakly compatible is STx=TSx whenever sx=Tx

Definition 2.4[1]A pair of self-maps A and S on a complex valued metric space (X,d) satisfy the property (E.A) if there exist a sequence $\{x_n\}$in X such that$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Lemma2.5[10]Three pairs of self-maps (A,Q),(S,T) and(B,P) on a complex valued metric space (X,d) satisfy common property (E.A) if there exists two sequences $\{x_n\}$,$\{z_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Py_n = \lim_{n\to\infty} Qy_n = p$ for some $p \in X$.

Definition 2.6[10] Two finite family of self-maps$\{A_i\}_{i=1}^{i=m}$ and $\{B_j\}_{j=1}^{n=m}$ on a set X are pairwise commuting if

i. $A_iA_j = A_jA_i$, $i,j \in \{1,2,3,4...m\}$,
ii. $B_iB_j = B_jB_i$, $i,j \in \{1,2,3,4\ldots n\}$,
iii. $A_iB_j = B_jA_i$, $j \in \{1,2,3,4\ldots.m\}, j \in \{1,2,3,4\ldots n\}$.

3. Main result

Implicit relations playconsiderable role in establishing of common fixed point results. Let M^{15} be the set of all continuous functions satisfying the following conditions:

a. $\emptyset(u,0,u,0,u,0,0,u,u,0,0,0,0,0,0) \precsim 0 \Rightarrow u \precsim 0$
b. $\emptyset(u,0,0,0,0,u,0,0,0,u,u,0,0,u,0) \precsim 0 \Rightarrow u \precsim 0$
c. $\emptyset(0,0,u,0,0,0,0,u,0,u,0,u,0,0,u) \precsim 0 \Rightarrow u \precsim 0$
d. $\emptyset(u,u,u,u,u,0,0,0,0,0,0,0,0,0,0) \precsim 0 \Rightarrow u \precsim 0$

Example 3.1:Define$\emptyset: \mathbb{C}^{15} \to \mathbb{C}$ as

$$\emptyset(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10},x_{11},x_{12},x_{13},x_{14},x_{15}\} = x_1 - \emptyset(\min\{x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10},x_{11},x_{12},x_{13},x_{14},x_{15}\})$$

Where $\emptyset_1: \mathbb{C} \to \mathbb{C}$ is icresing and continuous function such that $\emptyset_1$(y)>y

For all $y \in X$, clearly, $\emptyset$ satisfies all three conditions. therefore $\emptyset \in M^{15}$

obsevations:

Lemma 3.2: Let A,Q,S,T,B and P be self-mappings of a complex valued metric space (X,d) satisfying the following conditions:

(1.5) the pairs (A,Q),(B,P) ,(S,T) satisfies the property E.A.;

(1.6) for any x,y, z ∈X, ∅ ∈M_{15}'

$$\emptyset\begin{pmatrix} d(Ax,By),d(Ax,Py),d(Ax,Sz),d(Ax,Tz),d(Ax,Qx),\\ d(Qx,By),d(Qx,Py),d(Qx,Sz),d(Qx,Tz),d(By,Sz),d(By,Tz),\\ d(Py,Sz),d(Py,Tz),d(By,Py),d(Sz,Tz)\end{pmatrix}\precsim 0$$

(1.7)A(X)⊂ P(X)) ⊂ T(X) orB(X)⊂T(X) or S(X)⊂P(X)

Then the pairs (A,Q),(B,P) ,(S,T) share the common (E.A) property.

Proof

Suppose the pair (A,Q) and(S,T)satisfy property E.A., then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n$=$\lim_{n\to\infty} Qx_n$=p for some p∈ X. Since A(X)⊂ P(X) ⊂ T(X)hence for each $\{x_n\}$, $\{y_n\}$ in X there exist $\{z_n\}$ in X such that Px_n= $Ax_n = Tx_n$

Therefore $\lim_{n\to\infty} Ax_n$=$\lim_{n\to\infty} Px_n$=p= $\lim_{n\to\infty} Qy_n$=$\lim_{n\to\infty} Tx_n$ for some p∈ X. Since

Now we claim that $\lim_{n\to\infty} By_n$=p. suppose that $\lim_{n\to\infty} By_n$ ≠p. then applying inequality (1.6), we obtain

$$\emptyset\begin{pmatrix} d(Ax_n,By_n),d(Ax_n,Py_n),d(Ax_n,Sz_n),d(Ax_n,Tz_n),d(Ax_n,Qx_n),\\ d(Qx_n,By_n),d(Qx_n,Py_n),d(Qx_n,Sz_n),d(Qx_n,Tz_n),d(By_n,Sz_n),d(By_n,Tz_n),\\ d(Py_n,Sz_n),d(Py_n,Tz_n),d(By_n,Py_n),d(Sz_n,Tz_n)\end{pmatrix}\precsim 0$$

Taking n→ ∞, we obtain

$$\emptyset\begin{pmatrix} d\left(p,\lim_{n\to\infty} By_n\right),d(p,p),d(p,p),d(p,p),d(p,p),\\ d\left(p,\lim_{n\to\infty} By_n\right),d(p,p),d(p,p),d(p,p),d\left(\lim_{n\to\infty} By_n,p\right),\\ d\left(\lim_{n\to\infty} By_n,p\right),d(p,p),d(p,p),d\left(\lim_{n\to\infty} By_n,p\right),d(p,p)\end{pmatrix}\precsim 0$$

$$\emptyset\begin{pmatrix} d\left(z,\lim_{n\to\infty} By_n\right),0,0,0,0,d\left(p,\lim_{n\to\infty} By_n\right),0,0,0,\\ d\left(\lim_{n\to\infty} By_n,p\right),d\left(\lim_{n\to\infty} By_n,p\right),0,0,\\ d\left(p,\lim_{n\to\infty} By_n,p\right),0\end{pmatrix}\precsim 0$$

Which is a contradiction to using (b.),we get

$\mathrm{d}(\mathrm{z}, \lim_{n\to\infty} \mathrm{B}y_n) \lesssim 0$ which gives $|\mathrm{d}(\mathrm{p}, \lim_{n\to\infty} \mathrm{B}y_n)| \lesssim 0$,a contradiction and therefore,

$\lim_{n\to\infty} By_n$=p. hence the pair (A,Q) and(S,T) satisfy common (E.A) property.

Now, Suppose the pair (A,Q) and(B,P)satisfy property E.A., then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n$=$\lim_{n\to\infty} Qx_n$=p for some p$\in X$. Since A(X)⊂ P(X) ⊂ T(X) hence for each $\{x_n\}$, $\{y_n\}$ in X there exist $\{z_n\}$ in X such that Px_n= $Ax_n = Tx_n$

Therefore $\lim_{n\to\infty} Ax_n$=$\lim_{n\to\infty} Px_n$=p= $\lim_{n\to\infty} Qy_n$ =$\lim_{n\to\infty} Tx_n$ for some p$\in X$. Since

Now we claim that $\lim_{n\to\infty} Sz_n$=p. suppose that $\lim_{n\to\infty} Sz_n \neq$p. then applying inequality (1.6), we obtain

$$\emptyset\begin{pmatrix} \mathrm{d}(\mathrm{A}x_n, \mathrm{B}y_n), \mathrm{d}(\mathrm{A}x_n, \mathrm{P}y_n), \mathrm{d}(\mathrm{A}x_n, \mathrm{S}z_n), \mathrm{d}(\mathrm{A}x_n, \mathrm{T}z_n), \mathrm{d}(\mathrm{A}x_n, \mathrm{Q}x_n), \\ \mathrm{d}(\mathrm{Q}x_n, \mathrm{B}y_n), \mathrm{d}(\mathrm{Q}x_n, \mathrm{P}y_n), \mathrm{d}(\mathrm{Q}x_n, \mathrm{S}z_n), \mathrm{d}(\mathrm{Q}x_n, \mathrm{T}z_n), \mathrm{d}(\mathrm{B}y_n, \mathrm{S}z_n), \mathrm{d}(\mathrm{B}y_n, \mathrm{T}z_n), \\ \mathrm{d}(\mathrm{P}y_n, \mathrm{S}z_n), \mathrm{d}(\mathrm{P}y_n, \mathrm{T}z_n), \mathrm{d}(\mathrm{B}y_n, \mathrm{P}y_n), \mathrm{d}(\mathrm{S}z_n, \mathrm{T}z_n) \end{pmatrix} \lesssim 0$$

Taking n→ ∞, we obtain

$$\emptyset\begin{pmatrix} \mathrm{d}(\mathrm{p},\mathrm{p}), \mathrm{d}(\mathrm{p},\mathrm{p}), \mathrm{d}\left(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}z_n\right), \mathrm{d}(\mathrm{p},\mathrm{p}), \mathrm{d}(\mathrm{p},\mathrm{p}), \mathrm{d}(\mathrm{p},\mathrm{p}), \mathrm{d}(\mathrm{p},\mathrm{p}) \\ \mathrm{d}\left(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}y_n\right), \mathrm{d}(\mathrm{p},\mathrm{p}), \mathrm{d}(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}y_n), \mathrm{d}(\mathrm{p},\mathrm{p}), \mathrm{d}\left(\lim\limits_{n\to\infty} \mathrm{S}x_n, \mathrm{p}\right), \mathrm{d}(\mathrm{p},\mathrm{p}), \\ \mathrm{d}(\mathrm{p},\mathrm{p}), \mathrm{d}\left(\lim\limits_{n\to\infty} \mathrm{S}x_n, \mathrm{p}\right) \end{pmatrix} \lesssim 0$$

Taking n→ ∞, we obtain

$$\emptyset\begin{pmatrix} 0{,}0, \mathrm{d}\left(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}z_n\right), 0{,}0{,}0{,}0, \mathrm{d}\left(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}z_n\right), 0, \\ \mathrm{d}\left(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}z_n\right), 0, \mathrm{d}\left(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}z_n\right), 0{,}0, \mathrm{d}\left(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}z_n\right) \end{pmatrix} \lesssim 0$$

Which is a contradiction to using (B),we get

$\mathrm{d}\left(\mathrm{p}, \lim\limits_{n\to\infty} \mathrm{S}z_n\right) \lesssim 0$ which gives $|\mathrm{d}(\mathrm{p}, \lim_{n\to\infty} \mathrm{S}z_n)| \lesssim 0$,a contradiction and therefore,

$\lim_{n\to\infty} Sz_n$=p. hence the pair (A,Q) and(B,P) satisfy common (E.A) property.

Similarly we can prove that the pairs (B,P) and (S,T) satisfy common (E.A) property

hence the pairs (A,Q),(B,P) and (S,T) share the common (E.A.) property.

Theorem 3.1 Let A,B,S,T,P,Q be self-mappings of a complex valued metric space (X,d) satisfying the condition (1.6) and

(1.8) (A,Q),(B,P) and (S,T) share the common (E.A) property;

(1.9) Q(X),P(X) and T(X) are closed subsets of X.

Then the pairs (A,Q),(B,P) and (S,T) have a point of coincidence each. Moreover, A,B,S,T,P,Q have a unique common fixed pointif there exist a coincidence points of one of the pair in .Provided all the three pairs (A,Q),(B,P) ,(S,T) are weakly compatible.

Proof: In view of (1.8) $\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}By_n=\lim_{n\to\infty}Ty_n=\lim_{n\to\infty}Py_n=\lim_{n\to\infty}Qy_n=p$ for some p∈ X.since Q(X)and T(X) areclosed subset of X, and let p be the coincidence point of the pair (S,T)there existu∈ x such that,

Qu=p=Tu=Sualso we claim that Au=p if Au≠p then by (1.6), take x=u=z and y=y_n

$$\emptyset\begin{pmatrix} d(Au,By_n),d(Au,Py_n),d(Au,Su),d(Au,Tu),d(Au,Qu), \\ d(Qx,By_n),d(Qu,Py_n),d(Qx,Su),d(Qx,Tu),d(By_n,Su),d(By_n,Tu), \\ d(Py_n,Su),d(Py_n,Tu),d(By_n,Py_n),d(Su,Tu) \end{pmatrix} \precsim 0$$

Taking the limit as n→ ∞

$$\emptyset\begin{pmatrix} d(Au,p),d(Au,p),d(Au,Su),d(Au,Tu),d(Au,p), \\ d(Qu,By_n),d(Qu,Py_n),d(Qu,Su),d(Qu,Tu),d(By_n,Su),d(By_n,Tu), \\ d(Py_n,Su),d(Py_n,Tu),d(By_n,Py_n),d(Su,Tu) \end{pmatrix} \precsim 0$$

$$\emptyset\begin{pmatrix} d(Au,p),d(Au,p),d(Au,Su),d(Au,Tu),d(Au,p), \\ d(p,p),d(p,p),d(p,Su),d(p,Tu),d(p,Su),d(p,Tu), \\ d(p,Su),d(p,Tu),d(p,p),d(Su,Tu) \end{pmatrix} \precsim 0$$

$$\emptyset\begin{pmatrix} d(Au,p),d(Au,p),d(Au,p),d(Au,p),d(Au,p), \\ d(p,p),d(p,p),d(p,p),d(p,p),d(p,p),d(p,p), \\ d(p,p),d(p,p),d(p,p),d(p,p) \end{pmatrix} \precsim 0$$

$$\emptyset\begin{pmatrix} d(Au,p),d(Au,p),d(Au,Su),d(Au,Tu),d(Au,p), \\ 0,0,0,0,0,0, \\ 0,0,0,0 \end{pmatrix} \precsim 0$$

Using (d.), we get $d(Au,p) \leq 0$, a contradiction, therefore, Au=p=Qu which shows that u is a coincidence point of the pair (A,Q).

Since P(X) is also closed subset of X, therefore $\lim_{n\to\infty}Px_n$=u in P(X) and hence there existv∈ x , such that Pv=p=Au=Qu.also, let p be the coincidence point of the pair (S,T) there exist u∈ x such that$Pv = p = Au = Qu = Tu = Su$Now, we show that Bv=p also,

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then by using inequality (1.6), take x=u=z,y=v,we have

$$\emptyset\begin{pmatrix} d(Au,Bv), d(Au,Pv), d(Au,Su), d(Au,Tu), d(Au,Qu), \\ d(Qu,Bv), d(Qu,Pv), d(Qu,Su), d(Qu,Tu), d(Bv,Su), d(Bv,Tu), \\ d(Pv,Su), d(Pv,Tu), d(Bv,Pv), d(Su,Tu) \end{pmatrix} \precsim 0$$

Taking the limit as n→ ∞

$$\emptyset\begin{pmatrix} d(Au,p), d(Au,p), d(Au,Su), d(Au,Tu), d(Au,p), \\ d(p,p), d(p,p), d(p,Su), d(p,Tu), d(p,Su), d(p,Tu), \\ d(p,Su), d(p,Tu), d(p,p), d(Su,Tu) \end{pmatrix} \precsim 0$$

$$\emptyset\begin{pmatrix} d(p,Bv), d(p,p), d(p,p), d(p,p), d(p,p), \\ d(p,Bv), d(p,p), d(p,p), d(p,p), d(Bv,p), d(Bv,p), \\ d(p,p), d(p,p), d(Bv,p), d(p,p) \end{pmatrix} \precsim 0$$

$$\emptyset\begin{pmatrix} d(p,Bv), 0,0,0,0, \\ d(p,Bv), 0,0,0, d(Bv,p), d(Bv,p), \\ 0,0, d(Bv,p), 0 \end{pmatrix} \precsim 0$$

Using (b.), we get $d(Bv,p) \leq 0$, a contradiction, therefore, Bv=Pv=p which shows that v is a coincidence point of the pair (B,P).

Similarly we can prove that w is a coincidence point of (S,T) using above coincidence points. Since the paits (A,Q),(B,P) and (S,T) are weakly compatible and Au=Qu,Bv=Pv and Sw=Tw, therefore, Ap=AQu=QAu=Qu, Bp=BPp=PBp=Pv and Sw=STw=TSw=Tw.

If Ap≠ p then by using inequality (1.6), we have

$$\emptyset\begin{pmatrix} d(Ap,Bv), d(Ap,Pv), d(Ap,Sw), d(Ap,Tw), d(Ap,Qp), \\ d(Qp,Bv), d(Qp,Pv), d(Qp,Sw), d(Qp,Tw), d(Bv,Sw), d(Bv,Tw), \\ d(Pv,Sw), d(Pv,Tz), d(Bv,Pv), d(Sw,Tw) \end{pmatrix} \precsim 0$$

$$\emptyset\begin{pmatrix} d(Ap,p), d(Ap,p), d(Ap,p), d(Ap,p), d(Ap,p), \\ d(p,p), d(p,p), d(p,p), d(p,p), d(p,p), d(p,p), \\ d(p,p), d(p,p), d(p,p), d(p,p) \end{pmatrix} \precsim 0$$

Using (d.), we get d(Ap,p)≾0 which gives,|d(Ap,p)|≤0, a contradiction hence Ap=p=Qp. Similarly, one can prove that Bp=Pp=p and Sp=Tp=p , and p is common fixed point of A,B,S,T,P and Q.

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